DELHI PUBLIC SCHOOL, DURGAPUR
QUESTION BANK FOR BLOCK TEST - 1 (2018-19)
CLASS-XI
SUB:MATHEMATICS

SET, RELATION AND FUNCTION

1. Show that $n\{P[P(P(\phi))]\}=4$

2. If A and B are any two sets then prove that (i) $(A\cup B)' = A'\cap B'$ (ii) $(A\cap B)' = A'\cup B'$

3. Let A,B,C be the sets such that $A\cup B = A\cup C$ and $A\cap B = A\cap C$ .Show that B = C

4. In a group of children , 35 play football out of which 20 play football only, 22 play hockey;25 play cricket out of which 11 play cricket only . Out of these 7 play cricket and football but not hockey , 3 play football and hockey but not cricket and 12 play football and cricket both. How many play all the three games ? How many play cricket and hockey but not football ? How many play hockey only ? What is the total number of children in the group? (5,2,12,60)

5. In a group of 50 people , 35 speak Hindi , 25 speak both English and Hindi and all the people speak at least one of the two languages . How many people speak only English and not Hindi ? How many people speak English ? (15,40)

6. A market research group conducted a survey of 2000 consumers and reported that 1720 consumers liked product P_1 and 1450 consumers liked product P_2. What is the least number that must have liked both the products? (1170)

7. A survey shows that 63% of the Americans like cheese whereas 76 % like apples . If x% of Americans like both cheese and apples , find the value of x .

8. Let A = $\{1,2,3,......14\}$. Define a relation R from A to A by R = $\{(x,y): 3x-y=0, where x,y\in A\}$.
Write down its domain , co domain and range.

9. Let R be a relation on Z defined by R = $\{(a,b): a,b\in Z, a-b is an integer\}$. Find the domain and range of the integer

10. Let R be a relation on the set of whole numbers W defined by
R= $\{(x,y): 2x+3y=12, x,y\in W\}$. Find (i) R (ii) Domain of R (iii) Range of R (iv)R^{-1}

11. Let A = $\{1,2,3\}$ and R = $\{(a,b): |a^2-b^2|\le 5, a,b\in A\}$.Then write R as a set of ordered pairs.

12. Let f = $\{(1,1),(2,3),(0,-1),(-1,-3)\}$be a function described by the formula f(x) = ax+b
for some integers a,b . Determine a,b. (a = 2,b = -1)

13. If f is a real function defined by f(x) = $\frac{x-1}{x+1}$, then prove that f(2x) = $\frac{3f(x)+1}{f(x)+3}$

14. If for non - zero x , a f(x) +b f $\left(\frac{1}{x}\right)$ = $\frac{1}{x}$ - 5, where a $\neq$ b, then find f(x)

15. Let f be the subset of Z x Z ,defined by f= $\{(ab, a+b) : a, b \in Z\}$. Is f a function from Z to Z ?

Justify your answer.

15 . Find the domain of :

(i) f(x) = $\frac{x-1}{x-3}$ (ii) $\frac{x^2+2x+1}{x^2-8x+12}$ (iii) $\sqrt{9-x^2}$ (iv) $\sqrt{\frac{x-2}{3-x}}$ (v) $\frac{1}{\sqrt{x-|x|}}$

(vi) $\frac{1}{\sqrt{x-[x]}}$ (vii) $\frac{1}{\log_{10}(1-x)}+\sqrt{x+2}$ (viii) $\sqrt{\frac{1-|x|}{2-|x|}}$ (ix) $\sqrt{\frac{x+3}{(2-x)(x-5)}}$

(x) $\sqrt{x-3-2\sqrt{x-4}}-\sqrt{x-3+2\sqrt{x-4}}$ (xi) $\sqrt{5|x|-x^2-6}$

16. Find range : (i) f(x) = 1- $|x-2|$ (ii) $\frac{|x-4|}{x-4}$

17. Let f = $\left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R to R . Determine the range of f .

18. Let f(x) = x^2 and g(x) = 2x+1 be two real functions. Find (f+g)(x), (f-g)(x), (fg)(x), $\left(\frac{f}{g}\right)$(x)

TRIGONOMETRY

1. In a circle of diameter 40 cm, the length of a chord is 20 cm . Find the length of the minor arc corresponding to the chord. ($\frac{20\pi}{3}$ cm)

2. The angles of a triangle are in AP . The number of degrees in the least is to the number of radians in the greatest as 60 : π . Find the angles in degrees. (30, 60, 90)

3. A horse is tethered to a stake by a rope 30 m long . If the horse moves along the circumference of a circle always keeping the rope tight , far how far it will have gone when the rope has traced angle of 105^0. (55m)

4. Find the diameter of the sun in Km , supposing that it subtends an angle $32'$ at the eye of an observer. Given that the distance of the sun is 91x10^6 km. (847407.4km)

5. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15^0

6. Find in degrees the angle subtended at the centre of a circle of diameter 50 cm by an arc length 11 cm.

7. Find the value of sin (-1230^0)

8. Prove that $\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}=2$

9. Prove that cos 510^0 cos 330^0 +sin 390^0 cos 120^0 = -1

10. If sin A = $\frac{3}{5}$, cos B = - $\frac{12}{13}$, where A and B both lie in the second quadrant , find the value of sin (A+B)

11. Prove that sin (n+1)A sin(n+2)A + cos (n+1) A cos (n+2) A = cos A

12. If $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=\frac{-3}{2}$, prove that $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$

13. If $\tan\beta=\frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$, show that $\tan(\alpha-\beta)=(1-n)\tan\alpha$

14. If $\tan(\pi\cos\theta)=\cot(\pi\sin\theta)$, prove that $\cos(\theta-\frac{\pi}{4})=\pm\frac{1}{2\sqrt{2}}$

15. If $\sin\alpha+\sin\beta=a$ and $\cos\alpha+\cos\beta=b$, show that (i) $\cos(\alpha+\beta)=\frac{b^2-a^2}{b^2+a^2}$ (ii) $\sin(\alpha+\beta)=\frac{2ab}{a^2+b^2}$

16. If tan A = x tan B , prove that $\frac{\sin(A-B)}{\sin(A+B)}=\frac{x-1}{x+1}$

17. Prove that $\tan 8\theta-\tan 6\theta-\tan 2\theta=\tan 8\theta\tan 6\theta\tan 2\theta$

18. Find the maximum and minimum values of $12\cos\theta+5\sin\theta+4$

19. Prove that $\sin 10^\circ\sin 30^\circ\sin 50^\circ\sin 70^\circ=\frac{1}{16}$

20. Prove that sin A sin (60^0 – A) sin (60^0 +A) = $\frac{1}{4}$ sin3A

21. Prove that $\cos\alpha+\cos\beta+\cos\gamma+\cos(\alpha+\beta+\gamma)=4\cos\frac{\alpha+\beta}{2}\cos\frac{\beta+\gamma}{2}\cos\frac{\gamma+\alpha}{2}$

22. If $a\sin\theta=b\sin(\theta+\frac{2\pi}{3})=c\sin(\theta+\frac{4\pi}{3})$, prove that ab + bc + ca = 0

23. Show that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=2\cos\theta$

24. Prove that $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$

25. Prove that : $\cos^2 A + \cos^2(A + \frac{\pi}{3}) + \cos^2(A - \frac{\pi}{3}) = \frac{3}{2}$

26. Prove that : $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14} = \frac{1}{64}$

27. If $\tan\beta = \frac{\tan\alpha + \tan\gamma}{1 + \tan\alpha\tan\gamma}$, prove that $\sin 2\beta = \frac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha \sin 2\gamma}$

28. Prove that $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

29. If $\tan\frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\phi}{2}$, prove that $\cos\phi = \frac{\cos\theta - e}{1 - e\cos\theta}$

30. Prove that $\cot\frac{\pi}{24} = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$

31. If $\tan\frac{\theta}{2} = \sqrt{\frac{a-b}{a+b}}\tan\frac{\phi}{2}$, prove that $\cos\theta = \frac{a\cos\phi + b}{a + b\cos\phi}$

32. If α and β are distinct roots of $a\cos\theta + b\sin\theta = c$, prove that $\sin(\alpha+\beta) = \frac{2ab}{a^2 + b^2}$

33. If $2\tan\alpha = 3\tan\beta$, prove that $\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$

34. If $\sin\alpha = \frac{4}{5}$ and $\cos\beta = \frac{5}{13}$, prove that $\cos\frac{\alpha-\beta}{2} = \frac{8}{\sqrt{65}}$

35. Prove that $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ = 1$

36. **Solve the following equations:**

(i) $\cos\theta + \cos 2\theta + \cos 3\theta = 0$ (ii) $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$ (iii) $\cos\theta\cos 2\theta\cos 3\theta = \frac{1}{4}$

(iv) $\tan\theta + \tan 2\theta + \tan 3\theta = 0$ (v) $\sin\theta + \cos\theta = \sqrt{2}$ (vi) $2\sin^2 x + \sin^2 2x = 2$

(vii) 4sin x sin2xsin 4x = sin 3x (viii) $\tan\theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right) = 3$ (ix) $2\cos^2\theta + 3\sin\theta = 0$

(x) $4\cos\theta - 3\sec\theta = \tan\theta$

COMPLEX NUMBER

1. Evaluate : $1+i^2+i^4+i^6++i^8+\dots\dots+i^{20}$
2. Find the multiplicative inverse of z = 3-2i
3. Find the conjugates of $\frac{(3-i)^2}{2+i}$
4. Express in the form of a +ib: (i) $\left(-2-\frac{1}{3}i\right)^3$ (ii) $\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)}$

(iii) $\frac{1}{1-\cos\theta+2i\sin\theta}$ (iv) $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$

5. Find the real values of x and y , if $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$
6. Evaluate the following : $x^4+4x^3+6x^2+4x+9$, when $x=-1+i\sqrt{2}$
7. If $z_1=2-i, z_2=1+i$, find $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$
8. Find the real values of θ for which the complex number $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real.
9. Find the square root of (i) 8 – 15 i (ii) 1- i (iii) i
10. Find the modulus and argument of (i) $-2+2i\sqrt{3}$ (ii) $\frac{1+i}{1-i}$ (iii) $\frac{1+3i}{1-2i}$
11. Find the modulus and argument of the following complex numbers and convert them into polar form:

(i) $\frac{-16}{1+i\sqrt{3}}$ (ii) $\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ (iii) $\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$

12. If α and β are different complex numbers with $|\beta|=1$, find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$
13. If $a+ib=\frac{(x+i)^2}{2x^2+1}$, prove that $a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2}$
14. Find all non- zero complex numbers z satisfying $\bar{z}=iz^2$
15. If $z_1=2-i, z_2=-2+i$, find (i) $\text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$ (ii) $\text{Im}\left(\frac{1}{z_1\bar{z}_1}\right)$

QUADRATIC EQUATION

1. Solve by factorization method : $4x^2-12x+25=0$
2. Solve : (i) $21x^2-28x+10=0$ (ii) $\sqrt{2}x^2+x+\sqrt{2}=0$ (iii) $6x^2-17ix-12=0$
3. Solve : (i) $(2+i)x^2-(5-i)x+2(1-i)=0$ (ii) $x^2-(3\sqrt{2}-2i)x-\sqrt{2}i=0$

MATHEMATICAL INDUCTION

Prove by principle of mathematical induction.

1. $2+5+8+11+......+(3n-1)=\frac{1}{2}n(3n+1)$

2. $1.3+3.5+5.7+......+(2n-1)(2n+1)=\frac{n(4n^2+6n-1)}{3}$

3. $\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+........+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$

4. $1+\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+........+\frac{1}{1+2+3+4......+n}=\frac{2n}{n+1}$, for all $n\in N$

5. $7^{2n}+2^{3n-3}.3^{n-1}$ is divisible by 25 for all $n\in N$

6. $3^{2n+2}-8n-9$ is divisible by 8 for all $n\in N$

7. n(n+1) (n+5) is a multiple of 3 for all $n\in N$

8. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+........+\frac{1}{2^n}=1-\frac{1}{2^n}$

9. $11^{n+2}+12^{2n+1}$ is divisible by 133 for all $n\in N$

10. $n< 2^n$ for all $n\in N$

LINEAR INEQUATION

. Solve:

1.(i) $\frac{5x-2}{3}-\frac{7x-3}{5}>\frac{x}{4}$ (ii) $\frac{x+1}{x+2}\geq 1$ (iii) $\frac{4x+3}{2x-5}<6$ (iii) $\frac{x}{2x+1}\geq\frac{1}{4},\frac{6x}{4x-1}<\frac{1}{2}$

(iv) $\left|\frac{3x-4}{2}\right|\leq\frac{5}{12}$ (v) $\left|\frac{2x-1}{x-1}\right|>2$ (vi) $\frac{|x+2|-x}{x}<2$ (vii) $|x-1|+|x-2|\geq 4$

2. Solve the following systems of linear inequations graphically

(i) $2x+3y\leq 6,\ x+4y\leq 4,\ x\geq 0,\ y\geq 0$

(ii) $x+y\geq 1,\ 7x+9y\leq 63,\ x\leq 6,\ y\leq 5,\ x\geq 0,\ y\geq 0$

(iii) $3x+4y\ \leq 12,\ 4x+3y\leq 12,\ x\ \geq 0,\ y\geq 0$

STRAIGHT LINES

1. If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the line $\frac{1}{2}$, find the slope of the other line. (3, $-\frac{1}{3}$)

2. Find the value of x for which the points (x,-1), (2,1) and (4,5) are collinear . (1)

3. Find the angle between X – axis and the line joining the points (3,-1) and (4,-2). ($\frac{3\pi}{4}$)

4. Find the equation of the straight line which makes angle of 15^0 with the positive direction of x – axis and which cuts

an intercept of length 4 on the negative direction of Y – axis . $(y = (2-\sqrt{3})x - 4)$

5. Two lines passing through the point(2,3)intersect each other at an angle of 60^0 . If slope of one line is 2 , find the equation of the other line .

6. Find the equation of the line passing through the point (0,2) making an angle $\frac{2\pi}{3}$ with the positive x – axis. Also find the equation of the line parallel to it and crossing the y – axis at a distance of 2 units below the origin.

7. A line passing through the point A(3,0) makes 30^0 angle with the positive direction of x – axis. If this line is rotated through an angle of 15^0 in clockwise direction , find its equation in new position .

8. Find the equation of the internal bisector of angle BAC of the triangle ABC whose vertices A,B,C are (5,2),(2,3) and (6,5) respectively. (2x+y-12 = 0)

9. Find the equation of the line which passes through the point (3,4) and the sum of its intercepts on the axes is 14. (x+y = 7 , 4x +3y = 24)

10. A line passes through the point (3,-2) . Find the locus of the middle point of the portion of the line intercepted between the axes. (3y-2x =2xy)

11. Find the equation of the straight line upon which the length of the perpendicular from the origin is 5 and the slope of this perpendicular is $\frac{3}{4}$ (4x+3y -25 =0 , 4x+3y+25 = 0)

12. Find the equation of a straight line on which the perpendicular from the origin makes an angle of 30^0 with the x – axis and which forms a triangle of area $\frac{50}{\sqrt{3}}$ with the axes . $(\sqrt{3}x + y = \pm 10)$

13. Two vertices of a triangle are (3,-1) and (-2,3) and its orthocenter is at the origin . Find the co ordinates of the third vertex. $(\frac{-36}{7}, \frac{-45}{7})$

14. Prove that the lines 3x+y-14 = 0 , x-2y = 0 and 3x – 8y +4 = 0 are concurrent.

15. Find the image of the point (-8,12) with respect to the line mirror 4x+7y +13 =0. (-16,-2)

16. Prove that the straight lines (a+b)x +(a-b)y=2ab, (a-b)x +(a+b)y=2ab and x+y = 0 form an isosceles triangle whose vertical angle is $2\tan^{-1}\left(\frac{a}{b}\right)$

17. The hypotenuse of a right isosceles triangle has its ends at the points (1,3) and (-4,1).Find the equation of the legs (perpendicular sides) of the triangle. (7x -3y +31 = 0, 3x+7y -24 =0)

18. The equation of the base of an equilateral triangle is x+y-2 =0 and the opposite vertex has co ordinates (2,-1) . Find the area of the triangle. $\left(\frac{1}{2\sqrt{3}}\right)$ sq units

19. Prove that the length of perpendiculars from points P(m^2 , 2m) ,Q(mn,m+n) and R (n^2 , 2n) to the line $x\cos^2\theta + y\sin\theta\cos\theta + \sin^2\theta = 0$ are in GP

20. Show that the product of perpendiculars on the line $\frac{x}{a}cos\theta + \frac{y}{b}\sin\theta = 1$ from the points $\left(\pm\sqrt{a^2 - b^2}, 0\right)$ is b^2

21. If sum of perpendicular distances of a variable point P(x,y) from the lines x+y-5=0 and 3x-2y +7 = 0 is always 10 . Show that P must move on a line.

22. Prove that the line 5x-2y-1=0 is mid- parallel to the lines 5x-2y -9=0 and 5x-2y+7 = 0

23. Find the equations of the two straight lines through (7,9) and making an angle of 60^0 with the line $x-\sqrt{3}y-2\sqrt{3}=0$

24. A vertex of an equilateral triangle is (2,3) and the opposite side is x+y = 2 . Find the equations of the other sides.

25. One side of a rectangle lies along the line 4x+7y+5=0.Two of its vertices are (-3,1) and (1,1) .Find the equations of the other three sides.

CIRCLE

1. Find the equation of the circle which passes through the origin and cuts off intercepts 10 and 24 from the positive parts of x - and y- axis respectively. ($x^2+y^2-10x-24y=0$)
2. Find the equation of the circles which touch the axis of y at a distance 3 from the origin and cuts off an intercept of length 8 on the positive axis of x . ($x^2+y^2-10x\pm 6y+9=0$)
3. Find the equation of the circles whose centre (3,-1) and which cut off an intercept of length 6 from the line 2x-5y+18=0 ($x^2+y^2-6x+2y-28=0$)
4. Find the equation of the circle passing through the points (2,-6) , (6,4) and (-3,1)
($13x^2+13y^2-64x+10y-332=0$)
5. Find the equation of the circle passing through the points (2,3) and (-1,1) and whose centre is on the line x -3y -11 =0 ($x^2+y^2-7x+5y-14=0$)
6. Find the equation of the circle on the straight line joining the points of intersection of $x^2+y^2=25$ and $7x-y-25=0$ as diameter. ($x^2+y^2-7x+y=0$)
7. If 2x-y+6 =0 is a chord of the circle $x^2+y^2-2y-9=0$, find the equation of the circle with this chord as a diameter.
8. Find the equation of the circle which touches the lines 4x-3y +10 =0 and 4x-3y -30 =0 and whose centre lies on the line 2x+y =0 ($(x-1)^2+(y+2)^2=4^2$)
9. Find the equation of a circle of radius 5 which lies within the circle $x^2+y^2+14x+10y-26=0$ and which touches the given circle at the point (-1,3) ($(x+4)^2+(y+1)^2=5^2$)
10. Find the equation of the image of the circle $x^2+y^2+8x-16y+64=0$ in the line mirror x = 0.
($x^2+y^2-8x-16y+64=0$)

SYLLABUS FOR BLOCK TEST - 1

Sets , Relation , Function, Trigonometry , Trigonometric equations ,Complex numbers ,Quadratic Equation, Arithmetic progression , Geometric progression , Some special series, Mathematical Induction , Linear Inequation , straight lines , Circle